TRINITY COLLEGE

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	5	5	50	51	35
Section Two: Calculator-assumed	10	10	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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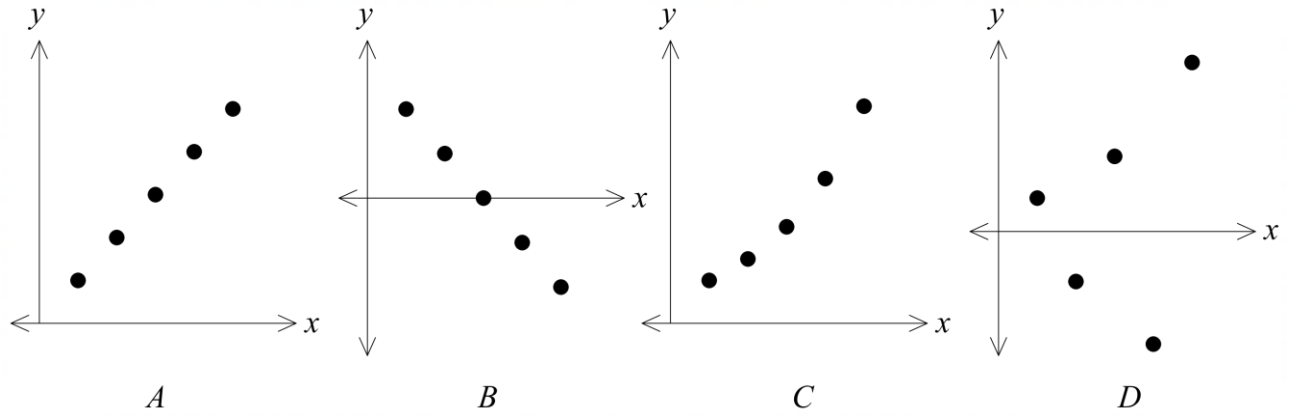
Section One: Calculator-free **35% (51 Marks)**

This section has **five** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Four graphs, A, B, C and D are drawn below.



State which of the graph, or graphs, above fit the following situations:

(a) There is a strong positive linear correlation between the two variables. (2 marks)

(b) 100% of the variation in y is explained by x. (2 marks)

(c) This graph has a correlation coefficient closest to zero. (1 mark)

(d) The graph shows a sequence with a negative common ratio. (1 mark)

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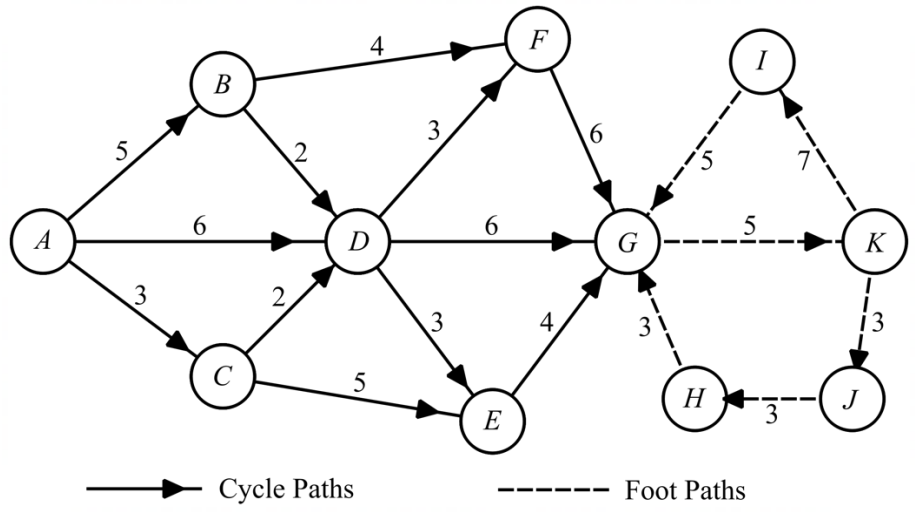
Question 2 **(12 marks)**

The network shows the cycle paths (indicated by a solid line) and footpaths (indicated by a dotted line) between various locations A to K in a city, and the time taken in minutes to travel along those paths.



George lives at A and needs to cycle to work at G.

(a) (i) Determine the path he should ride along in order to get to work in the shortest possible time. (2 marks)

(ii) State the time taken to ride this path. (1 mark)

George notices that at each intersection (B, C, D, E and F) he needs to wait, on average, 1.5 minutes for the traffic lights to change.

(b) Does the path George uses in part (a) still get him to work in the shortest possible time? Justify your answer by showing appropriate calculations, and stating any changes George should make. (3 marks)

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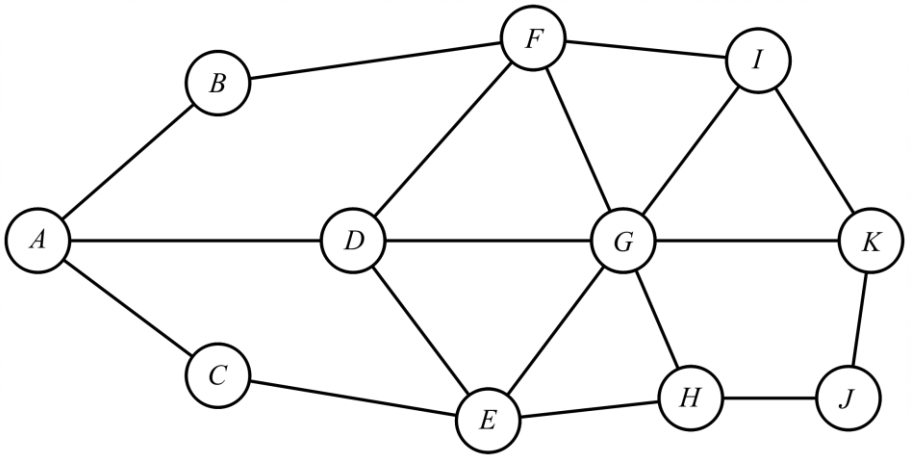
When George arrives at work, he notices he has a flat tyre. He decides to walk to a bicycle repair shop at K, and then get coffee at either Inca Café (at I) or Java Coffee (at J), before returning to work at G.

(c) (i) State the mathematical name of the route George will take. (1 mark)

(ii) Determine which coffee shop George should visit in order to complete these tasks in the shortest possible time. Justify your answer. (2 marks)

The network shows the roads between various locations, A to K.



(d) Is the network semi-Eulerian? Justify your answer. (2 marks)

(e) State an edge that would need to be added to make this network Eulerian. (1 mark)

Question 3 **(11 marks)**

A group of 10 students were asked to complete a multiple-choice quiz consisting of 10 questions.

The partially completed table below gives the number of hours of study completed by each student (x), the number of questions answered correctly (out of 10) (y), the predicted correct answers, and the residuals. Some of the data is missing.

Student	Tim	Joon	Jim	Kylie	Steve	Anne	Cindy	Anton	Lukas	Marie
Hours (x)	4	4.5	6	3.5	3	5	5.5	6.5	7	6.5
Correct Answers (y)	6	7		6	5	7	8	8	9	9
Predicted correct answers										
Residual	-0.1	0.45	0.1	0.35		0	0.55	-0.35	0.2	0.65

The least squares regression line for the data was modelled by the linear equation

$$\hat{y} = 0.9x + 2.5$$

(a) Anne has a residual of 0. Explain what this means. (1 mark)

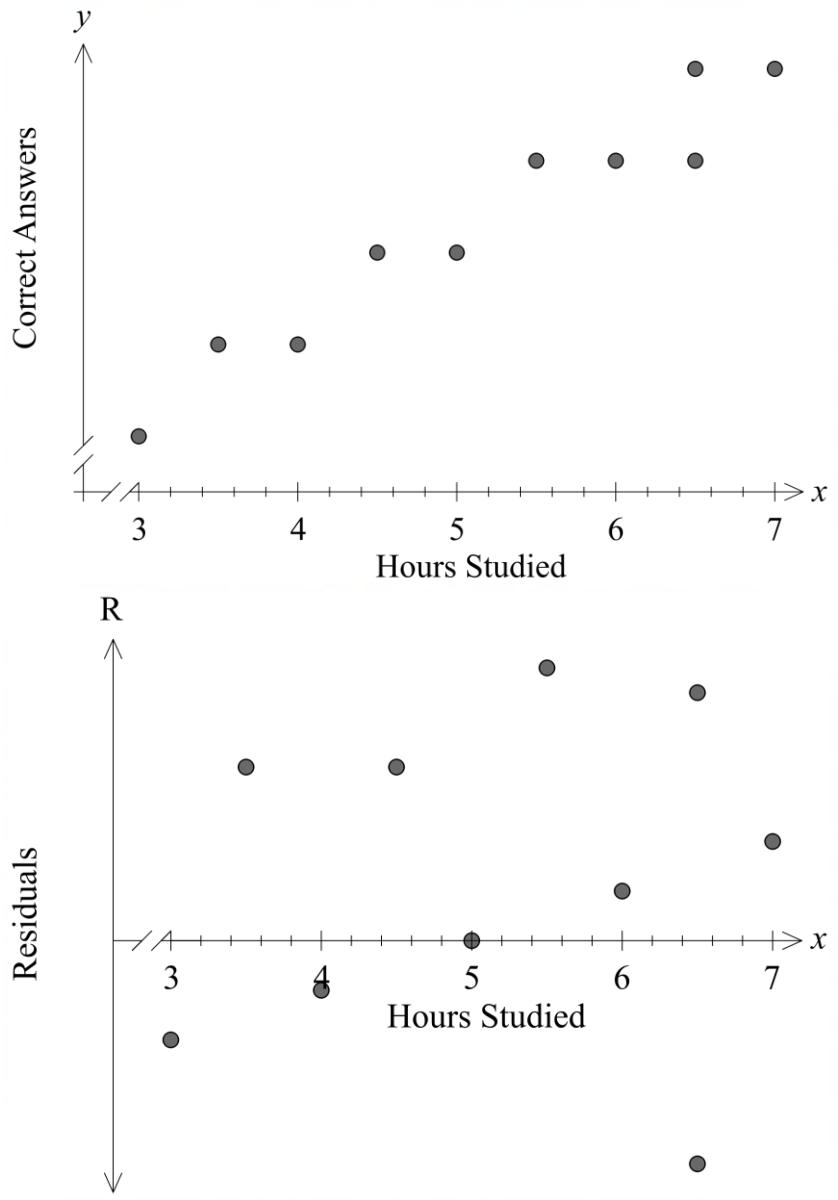
(b) Determine Steve's residual. (2 marks)

(c) Determine how many questions Jim answered correctly. (2 marks)

(d) How many students did better than predicted? Justify your answer. (2 marks)

See next page

A scatter plot of the data and a plot of the residuals are given below. The scale on the vertical axis has been removed on both of them.



(e) Describe the association between the hours studied, x, and the number of correct answers, y, in terms of direction and form. (2 marks)

(f) Does the residual plot indicate that the data is well modelled by a linear equation? Explain your answer. (2 marks)

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Question 4 **(12 marks)**

New Year 7 students at a school take part in a 'Greatest Race' event. In groups the students start at one of A, B, C or D. They answer clues that give them their next destination. Students must return to their starting point.

The adjacency matrix R (below) shows the order in which the students will travel.

$$R = \begin{array}{c} \\ \text{from} \end{array} \begin{array}{c} \begin{array}{cccc} \multicolumn{4}{c}{to} \\ A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The $'0'$ in row D, column A indicates that the clue at D does not direct them to A.
The $'1'$ in row D, column C indicates that the clue at D directs them to C.

A group of students start at A, and must return to A.

(a) (i) Assuming they answer the clues correctly, state the route that the students take. (2 marks)

(ii) If a network was drawn for the adjacency matrix, R, explain why it must be a directed graph. (1 mark)

(iii) Explain why any walks of length six in the network cannot be a trail. (2 marks)

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The adjacency matrix W (below) shows the number of ways of travelling between the various destinations.

$$W = \begin{matrix} \\ A \\ B \\ C \\ D \end{matrix} \begin{matrix} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{matrix}$$

(b) Determine the number of possible closed paths that the students in part (a)(i) can take. (1 mark)

(c) Give **two** reasons why the network represented by adjacency matrix, W, is not a simple graph. (2 marks)

The following matrices are calculated:

$$W^3 = \begin{bmatrix} 9 & 12 & 12 & 12 \\ 12 & 11 & 15 & 19 \\ 12 & 15 & 15 & 15 \\ 12 & 19 & 15 & 11 \end{bmatrix} \quad W + W^2 + W^3 = \begin{bmatrix} 12 & 16 & 16 & 16 \\ 16 & 17 & 20 & 23 \\ 16 & 20 & 20 & 20 \\ 16 & 23 & 20 & 17 \end{bmatrix}$$

(d) (i) Determine the number of walks of length 3 between B and D. (1 mark)

(ii) Describe why your answer to part (d)(i) does not tell you how many paths of length 3 there are between B and D. (1 mark)

(iii) Calculate the total number of walks of length 2 or 3 between A and C. (2 marks)

See next page

Question 5 **(10 marks)**

A house is built near a river bank. Each year the river floods and erodes the river bank.

At the start of 2020, the house is 2.5 m from the river bank. At the start of 2021, the river bank has eroded a total distance of 40 cm, and at the start of 2022 it has eroded a total distance of 60 cm.

(a) Assuming that the total distance eroded continues as an arithmetic sequence, determine

(i) the nth term of the sequence D_n, the total distance eroded n years after 2020, in the form $D_n = a + bn$. (2 marks)

(ii) in which year the river bank will reach the house. (3 marks)

At the start of 2023, the river bank had eroded a total distance of 90 cm.

(b) Explain why the arithmetic rule for D_n cannot be used to model this sequence. (1 mark)

A geometric rule with a ratio of 1.5 is required.

(c) (i) Determine a geometric recursive rule for T_n, the total distance eroded n years after 2020 showing clearly how the ratio is calculated. (2 marks)

(ii) In which year will the river bank now reach the house? (2 marks)

End of questions

Supplementary page

Question number: _____